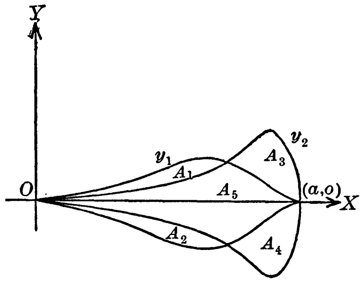
SOLUTION BY A. M. HARDING, University of Arkansas.

Since each of the curves is symmetrical with respect to both axes we shall consider only those areas which lie on the positive side of the x-axis.

$$A_1 = A_2 = \int_0^{a\sqrt{1/2}} (y_1 - y_2)dx = \frac{1}{a^4}\int_0^{a\sqrt{1/2}}$$

$$[x^2(a^2 - x^2)^{3/2} - x^4(a^2 - x^2)^{1/2}]dx$$

$$= \frac{1}{a^4}\int_0^{a\sqrt{1/2}} x^2\sqrt{a^2 - x^2}(a^2 - 2x^2)dx.$$



Let

$$x = a \sin\frac{\theta}{2}.$$

Then

$$A_1 = A_2 = \frac{a^2}{8}\int_0^{\pi/2} \sin^2\theta \cos\theta \, d\theta = \frac{a^2}{24}.$$

$$A_3 = A_4 = \int_{a\sqrt{1/2}}^{a} (y_2 - y_1)dx = \frac{1}{a^4}\int_{a\sqrt{1/2}}^{a} x^2\sqrt{a^2 - x^2}(2x^2 - a^2)dx.$$

Let $x^2 = a^2 - u^2$. Then

$$A_3 = A_4 = \frac{1}{a^4}\int_0^{a\sqrt{1/2}} u^2\sqrt{a^2 - u^2}(a^2 - 2u^2)du = \frac{a^2}{24},$$

as above.

$$\tfrac{1}{2}A_5 = \int_0^{a\sqrt{1/2}} y_2 dx + \int_{a\sqrt{1/2}}^{a} y_1 dx = \frac{1}{a^4}\int_0^{a\sqrt{1/2}} x^4\sqrt{a^2 - x^2}dx + \frac{1}{a^4}\int_{a\sqrt{1/2}}^{a} x^2(a^2 - x^2)^{3/2}dx.$$

The last integral reduces to

$$\frac{1}{a^4}\int_0^{a\sqrt{1/2}} u^4\sqrt{a^2 - u^2}du$$

on setting $x^2 = a^2 - u^2$. Hence,

$$\tfrac{1}{2}A_5 = \frac{2}{a^4}\int_0^{a\sqrt{1/2}} x^4\sqrt{a^2 - x^2}dx = \frac{a^2}{8}\left(\frac{\pi}{4} - \frac{1}{3}\right),$$

or

$$A_5 = \frac{a^2}{4}\left(\frac{\pi}{4} - \frac{1}{3}\right).$$

Also solved by A. R. NAUER, H. L. OLSON, and the Proposer.

2712 [June, 1918]. Proposed by WILLIAM HOOVER, Columbus, Ohio.

Given the conic $ax^2 + 2hxy + by^2 - 2x = 0$. Find the locus on which lie the four points of intersection of pairs of tangents to the conic from a pair of points on the x-axis equidistant from the origin.

SOLUTION BY A. H. WILSON, Haverford College.

The pair of tangents to the conic $C \equiv ax^2 + 2hxy + by^2 - 2x = 0$ from the point $(\alpha, 0)$ are represented by the equation $\lambda C + l^2 = 0$, where, $l \equiv (a\alpha - 1)x + h\alpha y - \alpha = 0$, is the polar of $(\alpha, 0)$, and $\lambda = \alpha(2 - a\alpha)$. Similarly, if $m \equiv (a\alpha + 1)x + h\alpha y - \alpha = 0$, $\mu = -\alpha(2 + a\alpha)$, $\mu C + m^2 = 0$, represents the tangents from $(-\alpha, 0)$.

The elimination of α from the equations $\lambda C + l^2 = 0$ and $\mu C + m^2 = 0$ is effected at once by subtraction and gives for the required locus the conic $hxy + by^2 - x = 0$.

As a does not occur in this equation, it is the locus described for any one of the conics of the one-parameter family obtained by holding h and b fixed and allowing a to vary.

Also solved by A. M. HARDING and the Proposer.